Second Edition

DERIVATIVES

PRINCIPLES and PRACTICE

Rangarajan K. Sundaram
Sanjiv R. Das

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To my lovely daughter Aditi
and
to the memory of my beautiful wife Urmilla

. . . RKS

To my departed parents
and
Priya and Shikhar

. . . SRD

Brief Contents

The following Web chapters are available at www.mhhe.com/sd2e:

Contents

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Author Biographies

Rangarajan K. ("Raghu") Sundaram is Professor of Finance at New York University's Stern School of Business. He was previously a member of the economics faculty at the University of Rochester. Raghu has an undergraduate degree in economics from Loyola College, University of Madras; an MBA from the Indian Institute of Management, Ahmedabad; and a Master's and Ph.D. in economics from Cornell University. He was co-editor of the *Journal of Derivatives* from 2002–2008 and is or has been a member of several other editorial boards. His research in finance covers a range of areas including agency problems, executive compensation, derivatives pricing, credit risk and credit derivatives, and corporate finance. He has also published extensively in mathematical economics, decision theory, and game theory. His research has appeared in all leading academic journals in finance and economic theory. The recipient of the Jensen Award and a finalist for the Brattle Prize for his research in finance, Raghu has also won several teaching awards including, in 2007, the inaugural Distinguished Teaching Award from the Stern School of Business. This is Raghu's second book; his first, a Ph.D.-level text titled *A First Course in Optimization Theory*, was published by Cambridge University Press.

Sanjiv Das is the William and Janice Terry Professor of Finance at Santa Clara University's Leavey School of Business. He previously held faculty appointments as associate professor at Harvard Business School and UC Berkeley. He holds post-graduate degrees in finance (M.Phil and Ph.D. from New York University), computer science (M.S. from UC Berkeley), an MBA from the Indian Institute of Management, Ahmedabad, B.Com in accounting and economics (University of Bombay, Sydenham College), and is also a qualified cost and works accountant. He is a senior editor of *The Journal of Investment Management*, co-editor of *The Journal of Derivatives* and the *Journal of Financial Services Research*, and associate editor of other academic journals. He worked in the derivatives business in the Asia-Pacific region as a vice-president at Citibank. His current research interests include the modeling of default risk, machine learning, social networks, derivatives pricing models, portfolio theory, and venture capital. He has published over eighty articles in academic journals, and has won numerous awards for research and teaching. He currently also serves as a senior fellow at the FDIC Center for Financial Research.

Preface

The two of us have worked together academically for more than a quarter century, first as graduate students, and then as university faculty. Given our close collaboration, our common research and teaching interests in the field of derivatives, and the frequent pedagogical discussions we have had on the subject, this book was perhaps inevitable.

The final product grew out of many sources. About three-fourths of the book was developed by Raghu from his notes for his derivatives course at New York University as well as for other academic courses and professional training programs at Credit Suisse, ICICI Bank, the International Monetary Fund (IMF), Invesco-Great Wall, J.P. Morgan, Merrill Lynch, the Indian School of Business (ISB), the Institute for Financial Management and Research (IFMR), and New York University, among other institutions. Other parts were developed by academic courses and professional training programs taught by Sanjiv at Harvard University, Santa Clara University, the University of California at Berkeley, the ISB, the IFMR, the IMF, and Citibank, among others. Some chapters were developed specifically for this book, as were most of the end-of-chapter exercises.

The discussion below provides an overview of the book, emphasizing some of its special features. We provide too our suggestions for various derivatives courses that may be carved out of the book.

An Overview of the Contents

The main body of this book is divided into six parts. Parts 1–3 cover, respectively, futures and forwards; options; and swaps. Part 4 examines term-structure modeling and the pricing of interest-rate derivatives, while Part 5 is concerned with credit derivatives and the modeling of credit risk. Part 6 discusses computational issues. A detailed description of the book's contents is provided in Section 1.5; here, we confine ourselves to a brief overview of each part.

Part 1 examines forward and futures contracts, The topics covered in this span include the structure and characteristics of futures markets; the pricing of forwards and futures; hedging with forwards and futures, in particular, the notion of *minimum-variance hedging* and its implementation; and interest-rate-dependent forwards and futures, such as forward-rate agreements or FRAs, eurodollar futures, and Treasury futures contracts.

Part 2, the lengthiest portion of the book, is concerned mainly with options. We begin with a discussion of option payoffs, the role of volatility, and the use of options in incorporating into a portfolio specific views on market direction and/or volatility. Then we turn our attention to the pricing of options contracts. The binomial and Black-Scholes models are developed in detail, and several generalizations of these models are examined. From pricing, we move to hedging and a discussion of the option "greeks," measures of option sensitivity to changes in the market environment. Rounding off the pricing and hedging material, two chapters discuss a wide range of "exotic" options and their behavior.

The remainder of Part 2 focuses on special topics: portfolio measures of risk such as Value-at-Risk and the notion of risk budgeting, the pricing and hedging of convertible bonds, and a study of "real" options, optionalities embedded within investment projects.

Part 3 of the book looks at swaps. The uses and pricing of interest rate swaps are covered in detail, as are equity swaps, currency swaps, and commodity swaps. (Other instruments bearing the "swaps" moniker are covered elsewhere in the book. Variance and volatility swaps are presented in the chapter on Black-Scholes, and credit-default swaps and total-return swaps are examined in the chapter on credit-derivative products.) Also included in Part 3 is a presentation of caps, floors, and swaptions, and of the "market model" used to price these instruments.

Part 4 deals with interest-rate modeling. We begin with different notions of the yield curve, the estimation of the yield curve from market data, and the challenges involved in modeling movements in the yield curve. We then work our way through factor models of the yield curve, including several well-known models such as Ho-Lee, Black-Derman-Toy, Vasicek, Cox-Ingersoll-Ross, and others. A final chapter presents the Heath-Jarrow-Morton framework, and also that of the Libor and swap market models.

Part 5 deals with credit risk and credit derivatives. An opening chapter provides a taxonomy of products and their characteristics. The remaining chapters are concerned with modeling credit risk. Structural models are covered in one chapter, reduced-form models in the next, and correlated-default modeling in the third.

Part 6, available online at `http://www.mhhe.com/sd1e`, looks at computational issues. Finite-differencing and Monte Carlo methods are discussed here. A final chapter provides a tutorial on the use of `Octave`, a free software program akin to `Matlab`, that we use for illustrative purposes throughout the book.

Background Knowledge

It would be inaccurate to say that this book does not presuppose any knowledge on the part of the reader, but it is true that it does not presuppose much. A basic knowledge of financial markets, instruments, and variables (equities, bonds, interest rates, exchange rates, etc.) will obviously help—indeed, is almost essential. So too will a degree of analytical preparedness (for example, familiarity with logs and exponents, compounding, present value computations, basic statistics and probability, the normal distribution, and so on). But beyond this, not much is required. The book is largely self-contained. The use of advanced (from the standpoint of an MBA course) mathematical tools, such as stochastic calculus, is kept to a minimum, and where such concepts are introduced, they are often deviations from the main narrative that may be avoided if so desired.

What Is Different about This Book?

It has been our experience that the overwhelming majority of students in derivatives courses go on to become traders, creators of structured products, or other users of derivatives, for whom a deep conceptual, rather than solely mathematical, understanding of products and models is required. Happily, the field of derivatives lends itself to such an end: while it is one of the most mathematically sophisticated areas of finance, it is also possible, perhaps more so than in any other area of finance, to explain the fundamental principles underlying derivatives pricing and risk-management in simple-to-understand and relatively non-mathematical terms. Our book looks to create precisely such a blended approach, one that is formal and rigorous, yet intuitive and accessible.

To this purpose, a great deal of our effort throughout this book is spent on explaining what lies behind the formal mathematics of pricing and hedging. How are forward prices determined? Why does the Black-Scholes formula have the form it does? What is the option gamma and why is it of such importance to a trader? The option theta? Why do term-structure models take the approach they do? In particular, what are the subtleties and pitfalls in modeling term-structure movements? How may equity prices be used to extract default risk of companies? Debt prices? How does default correlation matter in the pricing of portfolio credit instruments? Why does it matter in this way? In all of these cases and others throughout the book, we use verbal and pictorial expositions, and sometimes simple mathematical models, to explain the underlying principles before proceeding to a formal analysis.

None of this should be taken to imply that our presentations are informal or mathematically incomplete. But it is true that we eschew the use of unnecessary mathematics. Where discrete-time settings can convey the behavior of a model better than continuous-time settings, we resort to such a framework. Where a picture can do the work of a thousand (or even a hundred) words, we use a picture. And we avoid the presentation of "black box" formulae to the maximum extent possible. In the few cases where deriving the prices of some derivatives would require the use of advanced mathematics, we spend effort explaining intuitively the form and behavior of the pricing formula.

To supplement the intuitive and formal presentations, we make extensive use of numerical examples for illustrative purposes. To enable comparability, the numerical examples are often built around a common parametrization. For example, in the chapter on option greeks, a baseline set of parameter values is chosen, and the behavior of each greek is illustrated using departures from these baselines.

In addition, the book presents several full-length case studies, including some of the most (in)famous derivatives disasters in history. These include Amaranth, Barings, Long-Term Capital Management (LTCM), Metallgesellschaft, Procter & Gamble, and others. These are supplemented by other case studies available on this book's website, including Ashanti, Sumitomo, the Son-of-Boss tax shelters, and American International Group (AIG).

Finally, since the best way to learn the theory of derivatives pricing and hedging is by working through exercises, the book offers a large number of end-of-chapter problems. These problems are of three types. Some are conceptual, mostly aimed at ensuring the basic definitions have been understood, but occasionally also involving algebraic manipulations. The second group comprise numerical exercises, problems that can be solved with a calculator or a spreadsheet. The last group are programming questions, questions that challenge the students to write code to implement specific models.

New to this Edition

This edition has been substantially revised and incorporates many additions to and changes from the earlier one, entirely carried out by the first author. These include

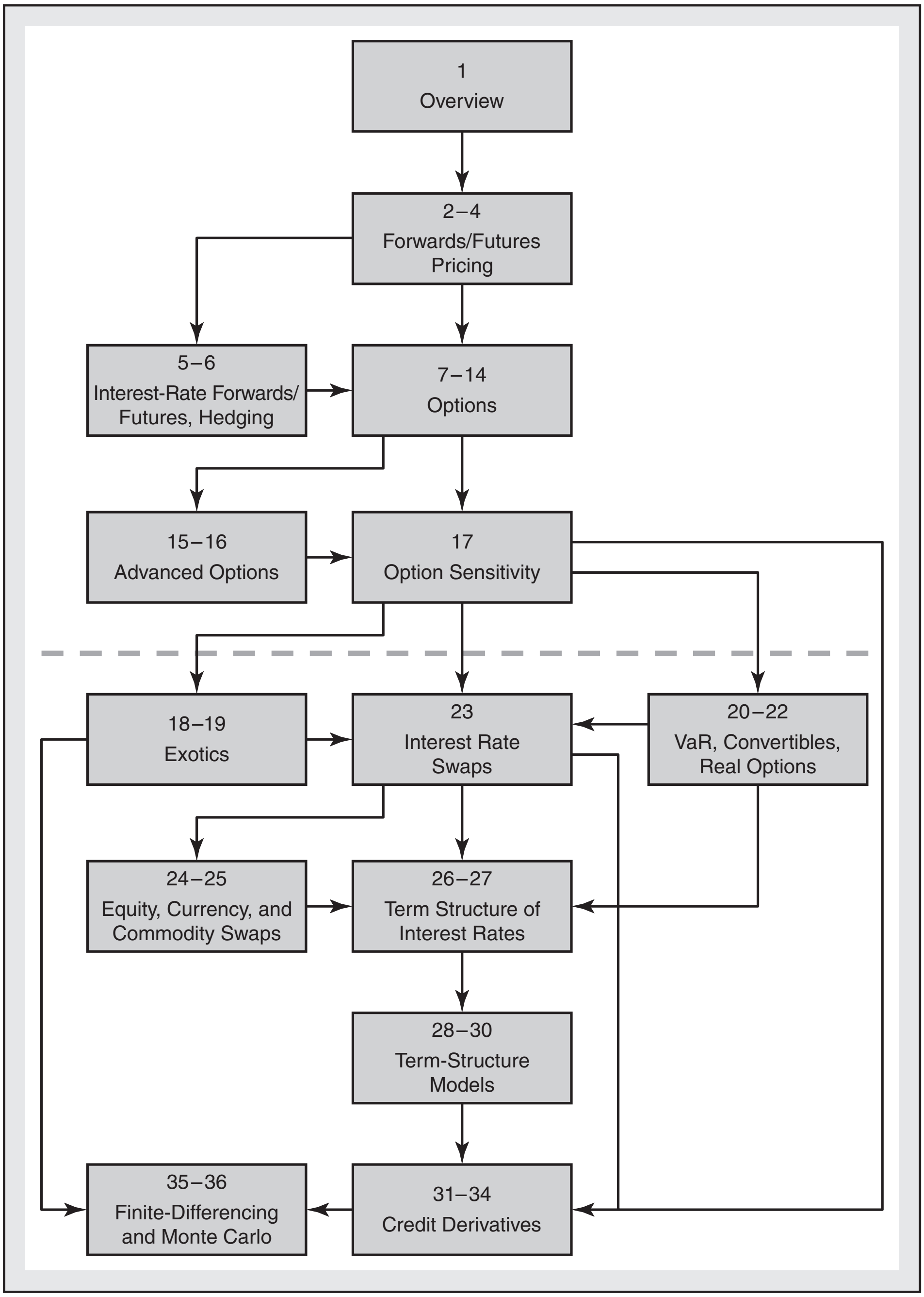
- brief to lengthy discussions of several new case studies (e.g., Aracruz Cellulose's $1 billion + losses from foreign-exchange derivatives in 2008, Société Générale's €5 billion losses from Jérôme Kerviel's "unauthorized" derivatives trading in 2008, Harvard University's $1.25 billion losses from swap contracts in 2009–13, the likely structure of the Goldman Sachs-Greece swap transaction of 2002 that allowed Greece to circumvent EU restrictions on debt, and others);
- expanded expositions of several key theoretical concepts (such the Black-Scholes formula in Chapter 14);
- detailed discussions of changing market practices (such as the new "dual curve" approach to swap pricing in Chapter 23 and the credit-event auctions that are hardwired into all credit-default swap contracts post-2009 in Chapter 31);
- new descriptions of exchange-traded instruments and indices (e.g., the CBoT's Ultra T-Bond futures in Chapter 6, the CBOE's BXM and BXY "covered call" indices in Chapter 8 or the CBOE's S&P 500 and VIX digital options in Chapter 18);
- and, of course, thanks to the assistance of students and colleagues, the identification and correction of typographical errors.

Special thanks to all those who sent in their comments and suggestions on the first edition. We trust the end-product is more satisfying.

Possible Course Outlines

Figure 1 describes the logical flow of chapters in the book. The book can be used at the undergraduate and MBA levels as the text for a first course in derivatives; for a second (or advanced) course in derivatives; for a "topics" course in derivatives (as a follow-up to a first course); and for a fixed-income and/or credit derivatives course; among others. We describe below our suggested selection of chapters for each of these.

FIGURE 1
The Flow of the Book



A first course in derivatives typically covers forwards and futures, basic options material, and perhaps interest rate swaps. Such a course could be built around Chapters 1–4 on futures markets and forward and futures pricing; Chapters 7–14 on options payoffs and trading strategies, no-arbitrage restrictions and put-call parity, and the binomial and Black-Scholes models; Chapters 17–19 on option greeks and exotic options; and Chapter 23 on interest rate swaps and other floating-rate products.

A second course, focused primarily on interest-rate and credit-risk modeling, could begin with a review of basic option pricing (Chapters 11–14), move on to an examination of more complex pricing models (Chapter 16), then cover interest-rate modeling (Chapters 26–30) and finally credit derivatives and credit-risk modeling (Chapters 31–34).

A "topics" course following the first course could begin again with a review of basic option pricing (Chapters 11–14) followed by an examination of more complex pricing models (Chapter 16). This could be followed by Value-at-Risk and risk-budgeting (Chapter 20); convertible bonds (Chapter 21); real options (Chapter 22); and interest-rate, equity, and currency swaps (Chapters 23–25), with the final part of the course covering either an introduction to term-structure modeling (Chapters 26–28) or an introduction to credit derivatives and structural models (Chapters 31 and 32).

Finally, a course on fixed-income derivatives can be structured around basic forward pricing (Chapter 3); interest-rate futures and forwards (Chapter 6); basic option pricing and the Black-Scholes model (Chapters 11 and 14); interest rate swaps, caps, floors, and swaptions, and the Black model (Chapter 23); and the yield curve and term-structure modeling (Chapters 26–30).

A Final Comment

This book has been several years in the making and has undergone several revisions in that time. Meanwhile, the derivatives market has itself been changing at an explosive pace. The financial crisis that erupted in 2008 will almost surely result in altering major components of the derivatives market, particularly in the case of over-the-counter derivatives. Thus, it is possible that some of the products we have described could vanish from the market in a few years, or the way these products are traded could fundamentally change. But the *principles* governing the valuation and risk-management of these products are more permanent, and it is those principles, rather than solely the details of the products themselves, that we have tried to communicate in this book. We have enjoyed writing this book. We hope the reader finds the final product as enjoyable.

Acknowledgments

We have benefited greatly from interactions with a number of our colleagues in academia and others in the broader finance profession. It is a pleasure to be able to thank them in print.

At New York University, where Raghu currently teaches and Sanjiv did his PhD (and has been a frequent visitor since), we have enjoyed many illuminating conversations over the years concerning derivatives research and teaching. For these, we thank Viral Acharya, Ed Altman, Yakov Amihud, Menachem Brenner, Aswath Damodaran, Steve Figlewski, Halina Frydman, Kose John, Tony Saunders, and Marti Subrahmanyam. We owe special thanks to Viral Acharya, long-time collaborator of both authors, for his feedback on earlier versions of this book; Ed Altman, from whom we—like the rest of the world—learned a great deal about credit risk and credit markets, and who was always generous with his time and support; Menachem Brenner, for many delightful exchanges concerning derivatives usage and structured products; Steve Figlewski, with whom we were privileged to serve as co-editors of the *Journal of Derivatives*, a wonderful learning experience; and, especially, Marti Subrahmanyam, who was Sanjiv's PhD advisor at NYU and with whom Raghu has co-taught executive-MBA and PhD courses on derivatives and credit risk at NYU since the mid-90s. Marti's emphasis on an intuitive understanding of mathematical models has considerably influenced both authors' approach to the teaching of derivatives; its effect may be seen throughout this book.

At Santa Clara University, George Chacko, Atulya Sarin, Hersh Shefrin, and Meir Statman all provided much-appreciated advice, support, and encouragement. Valuable input also came from others in the academic profession, including Marco Avellaneda, Pierluigi Balduzzi, Jonathan Berk, Darrell Duffie, Anurag Gupta, Paul Hanouna, Nikunj Kapadia, Dan Ostrov, N.R. Prabhala, and Raman Uppal. In the broader finance community, we have benefited greatly from interactions with Santhosh Bandreddi, Jamil Baz, Richard Cantor, Gifford Fong, Silverio Foresi, Gary Geng, Grace Koo, Apoorva Koticha, Murali Krishna, Marco Naldi, Shankar Narayan, Raj Rajaratnam, Rahul Rathi, Jacob Sisk, Roger Stein, and Ram Sundaram. The first author would particularly like to thank Ram Sundaram and Murali Krishna for numerous stimulating and informative conversations concerning the markets; the second author thanks Robert Merton for his insights on derivatives and guidance in teaching continuous-time finance, and Gifford Fong for many years of generous mentorship.

Over the years that this book was being written, many of our colleagues in the profession provided (anonymous) reviews that greatly helped shape the final product. A very special thanks to those reviewers who took the time to review virtually every chapter in draft form: Bala Arshanapalli (Indiana University–Northwest), Dr. R. Brian Balyeat (Texas A&M University), James Bennett (University of Massachusetts–Boston), Jinliang (Jack) Li (Northeastern University), Spencer Martin (Arizona State University), Patricia Matthews (Mount Union College), Dennis Ozenbas (Montclair State University), Vivek Pandey (University of Texas–Tyler), Peter Ritchken (Case-Western Reserve University), Tie Su (University of Miami), Thomas Tallerico (Dowling College), Kudret Topyan (Manhattan College), Alan Tucker (Pace University), Jorge Urrutia (Loyola University–Watertower), Matt Will (University of Indianapolis), and Guofu Zhou (Washington University–St. Louis).

As we have noted in the preface, this book grew out of notes developed by the authors for academic courses and professional training programs at a number of institutions including

Harvard University, Santa Clara University, University of California at Berkeley, Citibank, Credit-Suisse, Merrill Lynch, the IMF, and, most of all, New York University. Participants in all of these courses (and at London Business School, where an earlier version of Raghu's NYU notes were used by Viral Acharya) have provided detailed feedback that led to several revisions of the original material. We greatly appreciate the contribution they have made to the final product. We are also grateful to Ravi Kumar of Capital Metrics and Risk Solutions (P) Ltd. for his terrific assistance in creating the software that accompanies this book; and to Priyanka Singh of the same organization for proofreading the manuscript and its exercises.

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Chapter 1

Introduction

The world derivatives market is an *immense* one. The Bank for International Settlements (BIS) estimated that in June 2012, the total notional outstanding amount worldwide was a staggering $639 *trillion* with a combined market value of over $25 trillion (Table 1.1)—and this figure includes only over-the-counter (OTC) derivatives, those derivatives traded directly between two parties. It does not count the trillions of dollars in derivatives that are traded daily on the world's many exchanges. By way of comparison, world GDP in 2011 was estimated at just under $70 trillion.

For much of the last two decades, growth has been furious. Total notional outstanding in OTC derivatives markets worldwide increased almost *tenfold* in the decade from 1998 to 2008 (Table 1.2). Derivatives turnover on the world's exchanges quadrupled between 2001 and 2007, reaching a volume of over $2.25 *quadrillion* in the last year of that span (Table 1.3). Markets fell with the onset of the financial crisis, but by 2011–12, a substantial portion of that decline had been reversed.

The growth has been truly widespread. There are now thriving derivatives exchanges not only in the traditional developed economies of North America, Europe, and Japan, but also in Brazil, China, India, Israel, Korea, Mexico, and Singapore, among many other countries. A survey by the International Swaps and Derivatives Association (ISDA) in 2003 found that 92% of the world's 500 largest companies use derivatives to manage risk of various forms, especially interest-rate risk (92%) and currency risk (85%), but, to a lesser extent, also commodity risk (25%) and equity risk (12%). Firms in over 90% of the countries represented in the sample used derivatives.

Matching—and fueling—the growth has been the pace of innovation in the market. Traditional derivatives were written on commodity prices, but beginning with foreign currency and other financial derivatives in the 1970s, new forms of derivatives have been introduced almost continuously. Today, derivatives contracts reference a wide range of underlying instruments including equity prices, commodity prices, exchange rates, interest rates, bond prices, index levels, and credit risk. Derivatives have also been introduced, with varying success rates, on more exotic underlying variables such as market volatility, electricity prices, temperature levels, broadband, newsprint, and natural catastrophes, among many others.

This is an impressive picture. Once a sideshow in world financial markets, derivatives have today become key instruments of risk-management and price discovery. Yet derivatives have also been the target of fierce criticism. In 2003, Warren Buffet, perhaps the world's most successful investor, labeled them "financial weapons of mass destruction." Derivatives—especially credit derivatives—have been widely blamed for enabling, or at least exacerbating, the global financial markets crisis that began in late 2007. Victims of derivatives (mis-)use over the decades include such prominent names as the centuries-old British merchant bank Barings, the German industrial conglomerate Metallgesellschaft AG, the Japanese trading

TABLE 1.1 BIS Estimates of OTC Derivatives Markets Notional Outstanding and Market Values: 2008–12 (Figures in USD billions)

	Notional Outstanding			Gross Market Value		
	Jun. 2008	**Jun. 2010**	**Jun. 2012**	**Jun. 2008**	**Jun. 2010**	**Jun. 2012**
Total Contracts	**672,558**	**582,685**	**638,928**	**20,340**	**24,697**	**25,392**
FX Derivatives	**62,983**	**53,153**	**66,645**	**2,262**	**2,544**	**2,217**
Forwards and FX swaps	31,966	25,624	31,395	802	930	771
Currency swaps	16,307	16,360	24,156	1,071	1,201	1,184
Options	14,710	11,170	11,094	388	413	262
Interest-Rate Derivatives	**458,304**	**451,831**	**494,018**	**9,263**	**17,533**	**19,113**
Forward rate agreements	39,370	56,242	64,302	88	81	51
Interest rate swaps	356,772	347,508	379,401	8,056	15,951	17,214
Options	62,162	48,081	50,314	1,120	1,501	1,848
Equity Derivatives	**10,177**	**6,260**	**6,313**	**1,146**	**706**	**645**
Forwards and swaps	2,657	1,754	1,880	283	189	147
Options	7,521	4,506	4,434	863	518	497
Commodity Derivatives	**13,229**	**2,852**	**2,993**	**2,213**	**458**	**390**
Gold	649	417	523	72	45	62
Other commodities	12,580	2,434	2,470	2,141	413	328
Credit Derivatives	**57,403**	**30,261**	**26,931**	**3,192**	**1,666**	**1,187**
Single-name instruments	33,412	18,494	15,566	1,901	993	715
Multi-name instruments	23,991	11,767	11,364	1,291	673	472

Source: BIS website (`http://www.bis.org`).

powerhouse Sumitomo, the giant US insurance company, American International Group (AIG), and Brazil's Aracruz, then the world's largest manufacturer of eucalyptus pulp.

What *is* a derivative? What are the different types of derivatives? What are the benefits of derivatives that have fueled their growth? The risks that have led to disasters? How is the value of a derivative determined? How are the risks in a derivative measured? How can these risks be managed (or *hedged*)? These and other questions are the focus of this book. We describe and analyze a wide range of derivative securities. By combining the analytical descriptions with numerical examples, exercises, and case studies, we present an introduction to the world of derivatives that is at once formal and rigorous yet accessible and intuitive. The rest of this chapter elaborates and lays the foundation for the book.

What Are Derivatives?

A *derivative security* is a financial security whose payoff depends on (or *derives from*) other, more fundamental, variables such as a stock price, an exchange rate, a commodity price, an interest rate—or even the price of another derivative security. The underlying driving variable is commonly referred to as simply *the underlying*.

The simplest kind of derivative—and historically the oldest form, dating back thousands of years—is a *forward contract*. A forward contract is one in which two parties (commonly referred to as the *counterparties* in the transaction) agree to the terms of a trade to be consummated on a specified date in the future. For example, on December 3, a buyer and seller may enter into a forward contract to trade in 100 oz of gold in three months (i.e., on March 3) at a price of $1,500/oz. In this case, the seller is undertaking to sell 100 oz in three months at a price of $1,500/oz while the buyer is undertaking to buy 100 oz of gold in three months at $1,500/oz.

TABLE 1.2 BIS Estimates of OTC Derivatives Markets Notional Outstanding: 1998–2012 (Figures in USD billions)

	Jun 1998	Jun 2002	Jun 2006	Jun 2008	Jun 2010	Jun 2011	Jun 2012
Total Contracts	**72,134**	**127,509**	**372,513**	**672,558**	**582,685**	**706,884**	**638,928**
FX Derivatives	**18,719**	**18,068**	**38,127**	**62,983**	**53,153**	**64,698**	**66,645**
Forwards and FX swaps	12,149	10,426	19,407	31,966	25,624	31,113	31,395
Currency swaps	1,947	4,215	9,696	16,307	16,360	22,228	24,156
Options	4,623	3,427	9,024	14,710	11,170	11,358	11,094
Interest-Rate Derivatives	**42,368**	**89,955**	**262,526**	**458,304**	**451,831**	**553,240**	**494,018**
Forward rate agreements	5,147	9,146	18,117	39,370	56,242	55,747	64,302
Interest rate swaps	29,363	68,234	207,588	356,772	347,508	441,201	379,401
Options	7,858	12,575	36,821	62,162	48,081	56,291	50,314
Equity Derivatives	**1,274**	**2,214**	**6,782**	**10,177**	**6,260**	**6,841**	**6,313**
Forwards and swaps	154	386	1,430	2,657	1,754	2,029	1,880
Options	1,120	1,828	5,351	7,521	4,506	4,813	4,434
Commodity Derivatives	**443**	**777**	**6,394**	**13,229**	**2,852**	**3,197**	**2,993**
Gold	185	279	456	649	417	468	523
Other commodities	258	498	5,938	12,580	2,434	2,729	2,470
Forwards and swaps	153	290	2,188	7,561	1,551	1,846	1,659
Options	106	208	3,750	5,019	883	883	811
Credit Derivatives	–	–	**20,352**	**57,403**	**30,261**	**32,409**	**26,931**
Single-name instruments	–	–	13,873	33,412	18,494	18,105	15,566
Multi-name instruments	–	–	6,479	23,991	11,767	14,305	11,364
of which index products	–	–	–	–	7,500	12,473	9,731

Source: BIS website (`http://www.bis.org`).

A common motivation for entering into a forward contract is the elimination of cash-flow uncertainty from a future transaction. In our example, if the buyer anticipates a need for 100 oz of gold in three months and is worried about price fluctuations over that period, any uncertainty about the cash outlay required can be removed by entering into a forward contract. Similarly, if the seller expects to be offloading 100 oz of gold in three months and is concerned about prices that might prevail at the end of that horizon, entering into a forward contract locks in the price received for that future sale.

In short, forward contracts may be used as instruments of *hedging*. In financial parlance, hedging is the reduction in cash-flow risk associated with a market commitment. Forward contracts are commonly used by importers and exporters worried about exchange-rate fluctuations, investors and borrowers worried about interest-rate fluctuations, commodity producers and buyers worried about commodity price fluctuations, and so on.

A slightly more complex example of a derivative is an *option*. As in a forward, an option contract too specifies the terms of a future trade, but while a forward commits both parties to the trade, in an option, one party to the contract (called the *holder* of the option) retains the *right* to enforce or opt out of the contract. If the holder has the right to *buy* at the specified price, the option is called a *call option*; if the right to *sell* at that price, a *put option*.

The key difference between a forward and an option is that while a forward contract is an instrument for *hedging*, an option provides a form of financial *insurance*. Consider, for example, a call option on gold in which the buyer has the right to buy gold from the seller at a price of (say) $1,500/oz in three months' time. If the price of gold in three months is greater than $1,500/oz (for example, it is $1,530/oz), then the buyer will exercise the right in the contract and buy the gold for the contract price of $1,500. However, if the price in three months is less than $1,500/oz (e.g., is $1,480/oz), the buyer can choose to opt out of the contract and, if necessary, buy the gold directly in the market at the cheaper price of $1,480/oz.

Thus, holding a call option effectively provides the buyer with protection (or "insurance") against an *increase* in the price above that specified in the contract even while allowing the buyer to take full advantage of price decreases. Since it is the seller who takes the other side of the contract whenever the buyer decides to enforce it, it is the seller who provides this insurance to the buyer. In exchange for providing this protection, the seller will charge the buyer an up-front fee called the *call option premium*.

Analogously, a *put* option provides the seller with insurance against a *decrease* in the price. For instance, consider a put option on gold in which the seller has the right to sell gold to the buyer at $1,500/oz. If the price of gold falls below $1,500/oz, the seller can exercise the right in the put and sell the gold for $1,500/oz, but if the price of gold rises to more than $1,500/oz, then the seller can elect to let the put lapse and sell the gold at the higher market price. Holding the put insures the seller against a fall in the price below $1,500/oz. The buyer provides this insurance and will charge an up-front fee, the *put premium*, for providing this service.

Options offer an alternative to forwards for investors concerned about future price fluctuations. Unlike forwards, there is an up-front cost of buying an option (viz., the option premium) but, compensating for this, there is no compulsion to exercise if doing so would result in a loss.

Forwards and options are two of the most common and important forms of derivatives. In many ways, they are the building blocks of the derivatives landscape. Many other forms of derivatives exist, some which are simple variants of these structures, others much more complex or "exotic" (the favored term in the derivatives area for describing something that is not run-of-the-mill or "plain vanilla"). We elaborate on this later in this chapter and in the rest of the book. But first, we present a brief discussion on the different criteria that may be used to classify derivatives.

Classifying Derivatives

There are three popular ways to classify derivatives: by the underlying (equity, interest rate, etc.), by the nature of the instrument (forwards, futures, options, etc.), and by the nature of the market (over-the-counter versus exchange-traded).

A popular way to classify derivatives is to group them according to the underlying. For example, an *equity derivative* is one whose underlying is an equity price or stock index level; a *currency* or *FX* (short for foreign-exchange) *derivative* is one whose underlying is an exchange rate; and so on. Much of the world's derivatives trade on just a few common underlyings. Table 1.1 shows that *interest-rate derivatives* (derivatives defined on interest rates or on interest-rate-sensitive securities such as bonds) account for around 75% of the gross market value of the OTC derivatives market, with smaller shares being taken by currency, equity, commodity, and credit derivatives.

While these are the most common underlyings, derivatives may, in principle, be defined on just about any underlying variable. Indeed, a substantial chunk of the growth in derivatives markets in the early years of the 2000s came from *credit derivatives* (derivatives dependent on the credit risk of specified underlying entities), a category of derivatives that did not even exist in 1990. As noted earlier in this chapter, derivatives have also been introduced on a number of exotic underlying variables including electricity prices, temperature levels, broadband, newsprint, and market volatility.

A second popular way to classify derivatives is using the nature of instrument. Derivatives can differ greatly in the manner in which they depend on the underlying, ranging from very simple dependencies to very complex ones. Nonetheless, most derivatives fall into one of two classes: those that involve a *commitment* to a given trade or exchange of cash flows in

TABLE 1.3 BIS Estimates of Derivatives Turnover on Exchanges: 2001 to Q3–2012 (Figures in USD billions)

	2001	2004	2007	2009	2011	To Q3–2012
Futures Total	**445,683**	**830,546**	**1,584,611**	**1,126,519**	**1,524,141**	**898,253**
Interest rate	420,950	783,140	1,433,767	1,016,362	1,359,131	799,579
Currency	3,158	7,404	22,442	24,599	37,628	24,576
Equity index	21,575	40,001	128,401	85,559	127,382	74,098
North America	243,022	438,718	868,784	599,025	822,958	487,000
Europe	154,094	336,593	588,087	449,390	565,185	318,091
Asia and Pacific	43,394	48,547	107,907	63,125	107,490	72,875
Other markets	5,173	6,687	19,833	14,979	28,508	20,286
Options Total	**148,370**	**312,080**	**701,891**	**533,635**	**635,363**	**331,033**
Interest rate	122,766	260,056	547,629	434,601	466,281	254,255
Currency	356	589	2,141	1,980	2,525	1,776
Equity index	25,247	51,435	152,121	97,054	166,557	75,002
North America	107,684	181,503	414,728	216,390	262,515	158,288
Europe	33,473	101,954	203,787	258,557	258,265	125,692
Asia and Pacific	6,534	27,574	77,836	52,751	102,912	39,708
Other markets	679	1,048	5,540	5,936	11,671	7,345

Source: BIS website (`http://www.bis.org`).

the future and those in which one party has the *option* to enforce or opt out of the trade or exchange. Included in the former class are derivative securities such as *forwards*, *futures*; and *swaps*; derivatives in the latter class are called *options*.

Forwards and options have already been defined above. Futures contracts are similar to forward contracts except that they are traded on organized exchanges; we discuss the differences more precisely below. Swaps are contracts in which the parties commit to *multiple* exchanges of cash flows in the future, with the cash flows to be exchanged calculated under rules specified in the contract; thus, swaps are like forwards except with multiple transactions to which the parties commit.

Tables 1.1–1.3 use both of these schemes of classification, first breaking down the world derivatives market by underlying and then into forwards, futures, swaps, and options. The breakdown reveals some interesting variations. For example, while swaps account for the great bulk (roughly 80%) of OTC interest-rate derivatives, options constitute over 75% of OTC equity derivatives.

A third classification of derivatives is into over-the-counter (OTC) or exchange-traded derivatives. Over-the-counter derivatives contracts are traded bilaterally between two counterparties who deal directly with each other. In such transactions, each party takes the credit risk of the other (i.e., the risk that the other counterparty may fail to perform or "default" on the contract). In exchange-traded contracts, the parties deal though an organized exchange, and the identity of the counterparty is usually not known. Exchanges commonly guarantee performance on the contract, so each party is taking on only the credit risk of the exchange. Forwards and swaps are OTC contracts, while futures are exchange traded. Options can be both OTC and exchange traded.

1.1 Forward and Futures Contracts

A *forward contract* is an agreement between two parties to trade in a specified quantity of a specified good at a specified price on a specified date in the future. The following basic terminology is used when discussing these contracts:

- The buyer in the forward contract is said to have a *long position* in the contract; the seller is said to have a *short position*.
- The good specified in the contract is called the *underlying asset* or, simply, the *underlying*.
- The date specified in the contract on which the trade will take place is called the *maturity date* of the contract.
- The price specified in the contract for the trade is called the *delivery price* in the contract. This is the price at which delivery will be made by the seller and accepted by the buyer.

We will define the important concept of a *forward price* shortly. For the moment, we note that the forward price is related to, but is not the same concept as, the delivery price.

The underlying in a forward contract may be any commodity or financial asset. Forward contracts may be written on foreign currencies, bonds, equities, or indices, or physical commodities such as oil, gold, or wheat. Forward contracts also exist on such underlyings as interest rates or volatility which cannot be delivered physically (see, for example, the *forward-rate agreements* or FRAs described in Chapter 6, or the forward contracts on market volatility known as *variance* and *volatility swaps,* described in Chaper 14); in such cases, the contracts are settled in cash with one side making a payment to the other based on rules specified in the contract. Cash settlement is also commonly used for those underlyings for which physical delivery is difficult, such as equity indices.

As has been discussed, a primary motive for entering into a forward contract is *hedging*: using a forward contract results in locking-in a price today for a future market transaction, and this eliminates cash-flow uncertainty from the transaction. Foreign currency forwards, for example, enable exporters to convert the payments received in foreign currency into home currency at a fixed rate. Interest-rate forwards such as FRAs enable firms to lock-in an interest rate today for a future borrowing or investment. Commodity forwards such as forwards on oil enable users of oil to lock-in prices at which future purchases are made and refiners of oil to lock-in a price at which future sales are made.

Forward contracts can also be used for *speculation*, that is, without an underlying exposure already existing. An investor who feels that the price of some underlying is likely to increase can speculate on this view by entering into a long forward contract on that underlying. If prices do go up as anticipated, the investor can buy the asset at the locked-in price on the forward contract and sell at the higher price, making a profit. Similarly, an investor wishing to speculate on falling prices can use a short forward contract for this purpose.

Key Characteristics of Forward Contracts

Four characteristics of forward contracts deserve special emphasis because these are exactly the dimensions along which forwards and futures differ:

- First, a forward contract is a bilateral contract. That is, the terms of the contract are negotiated directly by the seller and the buyer.
- Second, a forward contract is customizable. That is, the terms of the contract (maturity date, quality of the underlying, etc.) can be "tailored" to the needs of the buyer and seller.
- Third, there is possible default risk for both parties. Each party takes the risk that the other may fail to perform on the contract.
- Fourth, neither party can transfer its rights and obligations in the contract unilaterally to a third party.

We return to these characteristics when discussing futures contracts.

TABLE 1.4 The Payoffs from a Forward Contract

This table describes the payoff to the long and short positions on the maturity date T of a forward contract with a delivery price of 100. S_T is the price of the underlying asset on date T.

Time-T Price S_T	Payoff to Long	Payoff to Short
⋮	⋮	⋮
70	−30	+30
80	−20	+20
90	−10	+10
100	–	–
110	+10	−10
120	+20	−20
130	+30	−30
⋮	⋮	⋮

Payoffs from Forward Contracts

The payoff from a forward contract is the profit or loss made by the two parties to the contract. Consider an example. Suppose a buyer and seller enter into a forward contract on a stock with a delivery price of $F = 100$. Let S_T denote the price of the stock on the maturity date T. Then, on date T,

- The long position is buying for $F = 100$ an asset worth S_T. So the payoff to the long position is $S_T - 100$. The long position makes a profit if $S_T > 100$, but loses if $S_T < 100$.
- The short position is selling for $F = 100$ an asset worth S_T. So the payoff to the short position is $100 - S_T$. The short position makes a profit if $S_T < 100$, but loses if $S_T > 100$.

For example:

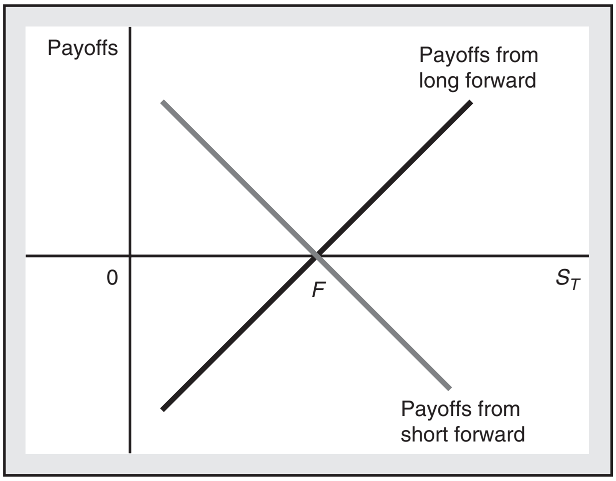
- If $S_T = 110$, then the long is buying for 100 an asset worth 110, so gains 10, but the short is selling for 100 an asset worth 110, so loses 10.
- If $S_T = 90$, the long is buying for 100 an asset worth only 90, so loses 10, while the short is selling for 100 an asset worth only 90, so gains 10.

Table 1.4 describes the payoff to the two sides for some other values of S_T. Two points about these payoffs should be noted. First, forwards (like all derivatives) are *zero-sum* instruments: the profits made by the long come at the expense of the short, and vice versa. The sum of the payoffs of the long and short is always zero. This is unsurprising. Except when the delivery price F exactly coincides with the time-T price S_T of the underlying, a forward contract involves an *off-market* trade (i.e., a trade at a different price from the prevailing market price). In any off-market trade, the benefit to one side is exactly equal to the loss taken by the other.

Second, as Figure 1.1 illustrates, forwards are “linear” derivatives. Every $1 increase in the price S_T of the underlying at date T increases the payoff of the long position by $1 and reduces the payoffs of the short position by $1. Linearity is a consequence of committing to the trade specified in the contract. In contrast, as we will see, options, which are characterized by their “optionality” concerning the trade, are fundamentally nonlinear instruments, and this makes their valuation and risk management much trickier.

FIGURE 1.1
Forwards Are Linear Derivatives

The figure shows the payoffs to the long and short positions on the maturity date T of a forward contract with delivery price F as the time-T price S_T of the underlying asset varies.



What Is the "Forward Price"?

By convention, neither party pays anything to enter into a forward contract. So the delivery price in the contract must be set so that the contract has zero value to both parties. This "breakeven" delivery price is called the *forward price*.

Is the forward price a well-defined concept? That is, is it obvious that there is only *one* breakeven delivery price? At first glance, it appears not. Certainly, it is true that if the delivery price is set very high, the short will expect to profit from the contract and the long to lose; that is, the contract will have positive value to the short and negative value to the long. Similarly, if the price is set too low, the contract will have positive value to the long (who will expect to profit from having access to the asset at an excessively low price) and negative value to the short. But it is not obvious that between these extremes, there is only one possible breakeven delivery price at which both parties will agree the contract has zero value. Intuitively, it appears that such idiosyncratic factors as risk-aversion and outlooks concerning the market ought to matter.

In Chapter 3, we examine this issue. We show that under fairly general conditions, the forward price *is*, in fact, a well-defined concept and that regardless of attitudes to risk and other factors, everyone *must* agree on the breakeven delivery price. Possible violations of these conditions and their consequences for the pricing theory are examined in Chapter 4. The principal assumption we make there, and throughout this book, is that markets do not permit *arbitrage*. The no-arbitrage assumption is just the minimal requirement that identical assets or baskets of assets trade at identical prices.

Futures Markets

A *futures contract* is, in essence, a forward contract that is traded on an organized exchange. But while futures and forwards are functionally similar (i.e., they serve the same economic purpose), the involvement of the exchange results in some important differences between them. We summarize the differences here; Chapter 2 deals with futures markets in detail.

First, in a futures contract, buyers and sellers deal through the futures exchange, not directly. The counterparties are unlikely to know each other's identities.

TABLE 1.5
Differences between Forwards and Futures

Criterion	Futures	Forwards
Buyer-seller interaction	Via exchange	Direct
Contract terms	Standardized	Can be tailored
Unilateral reversal	Possible	Not possible
Default-risk borne by	Exchange	Individual parties
Default controlled by	Margin accounts	Collateral

Second, because buyers and sellers do not meet, futures contracts must be *standardized*. Standardization covers the set of possible delivery dates and delivery locations, the size of one contract, and the quality or grade of the underlying that may be delivered under the contract, and is one of the most important functions performed by the exchange.

Third, counterparties are not exposed to each other's default risk. Rather, the exchange (or, more precisely, the clearinghouse, as we explain in Chapter 2) interposes itself between buyer and seller and guarantees performance on the contracts. Thus, each party to a futures transaction is exposed only to the default risk of the exchange. In well-run futures exchanges, this risk is generally very low. For example, no US exchange has ever defaulted on its obligations.

Fourth, an investor may, at any time, close out or *reverse* a futures position. Closing out involves taking an opposite position to the original one. For example, if the investor was initially long 10 futures contracts in gold for delivery in March, closing out involves taking short positions in 10 futures contracts for delivery in March. These positions are netted against each other, and, as far as the exchange is concerned, the investor has no net obligations remaining.

Fifth, having guaranteed performance on the futures contracts, the exchange must put safeguards in place to ensure it is not called upon to honor its guarantee too often. That is, it must ensure that the parties to the contract do not default in the first place. For this purpose, a system based on the use of "margin accounts" (a.k.a. "performance bonds") is commonly used.

Table 1.5 summarizes these main differences between futures and forwards. The institutional features of futures markets are designed to enhance the integrity and liquidity of the market, thereby making it more attractive to participants. However, they also have economic consequences. For example, futures prices—the breakeven delivery prices for futures contracts—are typically close to, but do not quite coincide with, forward prices because of these differences, as Chapter 3 discusses.

1.2 Options

An *option* is a financial security that gives its holder the right to buy or sell a specified quantity of a specified underlying asset at a specified price on or before a specified date. This simple definition contains several points of note.

First, an option gives the holder the right to buy or the right to sell (but not both!). The former class of options are called *call* options, the latter *put* options. The price that is specified in the contract at which the right may be availed of is called the *strike* or *exercise* price of the option, and the date on which the right expires, the *maturity* or *expiration* date of the option.

Second, options come in two basic "styles." In an *American*-style (or simply, American) option, the right specified the option can be exercised at any time on or before the option expiration. In a *European*-style option, the right can only be availed of *on* the maturity date. American options are generally more valuable than their European counterparts, though the two may sometimes have the same value (see Chapter 9).